

Chance-constrained Coordinated Optimization for Urban Electricity and Heat Networks

Zhinong Wei, Juan Sun, Zhoujun Ma, Guoqiang Sun, *Member, IEEE*, Haixiang Zang, Sheng Chen, *Student Member, IEEE*, Side Zhang and Kwok W. Cheung, *Fellow, IEEE*

Abstract—Urban electricity and heat networks (UEHN) consist of the coupling and interactions between electric power systems and district heating systems, in which the geographical and functional features of integrated energy systems are demonstrated. UEHN have been expected to provide an effective way to accommodate the intermittent and unpredictable renewable energy sources, in which the application of stochastic optimization approaches to UEHN analysis is highly desired. In this paper, we propose a chance-constrained coordinated optimization approach for UEHN considering the uncertainties in electricity loads, heat loads, and photovoltaic outputs, as well as the correlations between these uncertain sources. A solution strategy, which combines the Latin Hypercube Sampling Monte Carlo Simulation (LHSMCS) approach and a heuristic algorithm, is specifically designed to deal with the proposed chance-constrained coordinated optimization. Finally, test results on an UEHN comprised of a modified IEEE 33-bus system and a 32-node district heating system at Barry Island have verified the feasibility and effectiveness of the proposed framework.

Index Terms—Chance-constrained optimal power flow (CCOPF), correlated uncertainties, combined Latin Hypercube Sampling Monte Carlo Simulation (LHSMCS) approach and heuristic algorithm, urban electricity and heat networks (UEHN).

I. INTRODUCTION

AN integrated energy system (IES), which would hopefully break through the traditional independent planning and operations between different energy system sectors, is of great significance in improving energy efficiency, promoting large-scale development of renewable energy sources, and the realization of energy-conservation and emission-reduction targets. IES has become an important strategic research direction in the international energy arena [1]–[3]. Typically, IES encompasses various energy systems including electricity, gas, heating, cooling, hydrogen supply and electrified transportation systems, and the related communications and information

infrastructures [4]. With the widespread application of heating networks, the increasing penetration of combined heat and power (CHP) units and promising usage of power to heat (P2H) technology, the coupling and interactions between an electric power system (EPS) and district heating system (DHS) are becoming stronger and also accelerating in development. As a result, a regional IES represented by urban electricity and heat networks (UEHN) has developed rapidly in recent years. For example, studies on UEHN steady-state model, multi-energy flow calculation, and state estimation have been conducted in [5]–[10].

In the area of coordinated optimization for UEHN, considerable studies have been performed. Reference [11] proposed the dispatch model for heat and power production that uses electric boilers and heat storage tanks to enhance the flexibility of CHP units. A combined heat and power dispatch (CHPD) model, considering the heat storage characteristics of the pipeline of the heating network, was employed in [12]. The optimal operation of the integrated electrical and heating system in urban areas was established in [13], which indicates that the coordinated optimization of an electricity and heat network can effectively reduce the wind curtailment, fuel oil consumption, and heat losses. However, the above-mentioned studies [11]–[13] are confined to deterministic models without taking the impact of uncertainties into consideration.

Given the further expansion of installed capacity of renewable energy sources such as photovoltaic (PV), the fluctuant and intermittent nature of renewable generations pose great challenges to conventional deterministic optimizations. As such, employment of stochastic optimization approaches considering the influence of uncertainties is highly desired. Reference [14] proposed a scenario-based method considering the uncertainties of wind power outputs in a CHPD model but the constraints of the electricity and heat networks were ignored. A robust CHPD model was established and a decentralized solution based on Benders decomposition was proposed in [15], wherein only the uncertainties of wind power outputs were considered. In [16], uncertainties of both PV outputs and electricity loads were described by intervals, and an interval linear programming model was proposed to provide day-ahead optimal dispatch strategy for multiple integrated community energy systems. However, this model did not take the correlations of uncertain sources into account.

Introduced by Charnes and Cooper in 1959 [17], chance-constrained programming has become a competitive method

Manuscript received February 4, 2018; revised April 17, 2018; accepted April 24, 2018; date of publication December 30, 2018; date of current version October 30, 2018. This work was supported in part by Natural Science Foundation of Jiangsu Province, China (No. BK20171433) and in part by Science and Technology Project of State Grid Jiangsu Electric Power Corporation, China (No. J2018066).

Z. N. Wei, J. Sun (corresponding author, e-mail: hhusunjuan@163.com), G. Q. Sun, H. X. Zang, S. Chen, and S. D. Zhang are with the College of Energy and Electrical Engineering, Hohai University, Nanjing 210098, China.

Z. J. Ma is with the State Grid Jiangsu Electric Power Corporation, Nanjing 210098, China.

K. W. Cheung is with the GE Grid Solutions, Redmond, WA 98052, USA.
DOI: 10.17775/CSEEJPES.2018.00120

for solving optimization problems under uncertainties. Chance-constrained optimal power flow (CCOPF) requires that the inequality constraints can be satisfied at a certain confidence level. More specifically, violation of inequality constraints is allowed under some extreme scenarios. Consequently, additional economic benefits can be gained because the control actions for the extreme scenarios that rarely happen can be avoided. By using CCOPF, the optimality and reliability of system operations can be quantitatively balanced [18]. CCOPF has been widely used in EPS expansion planning, economic dispatch and unit commitment [19]. However, applications of CCOPF within UEHN are rather limited.

A combined MCS/analytical probabilistic power flow analysis method was proposed and a genetic algorithm was employed to accelerate the CCOPF process for EPS in [20]. However, the required computational burden may be huge. The heuristic algorithm based on cumulant methods has been employed for the CCOPF in EPS, in which the cross-cumulant approach is employed to address the correlated uncertainties and the calculation may be complicated [21].

In this paper, we develop a CCOPF formulation for UEHN, considering uncertain electricity and heat loads and PV outputs. The correlations between these uncertain sources are also included. Furthermore, a solution strategy, which combines the Latin Hypercube Sampling Monte Carlo simulation (LHSMCS) approach and a heuristic algorithm, is specially designed. More specifically, an optimal scheduling strategy that corresponds to the deterministic case is first obtained. Then, the LHSMCS approach and Cornish-Fisher series approximation are employed to perform probabilistic power and heat flow analysis, which provides the distributions of UEHN state variables and the constraint violation probabilities. After that, the corresponding upper and lower bounds are adjusted according to a heuristic algorithm if the violation probabilities exceed the allowed confidence level. The iteration is terminated until the obtained deterministic optimal scheduling strategy satisfies all chance constraints. Finally, the proposed chance-constrained framework is verified by UEHN comprised of a modified IEEE 33-bus system and a 32-node DHS at Barry Island.

The remainder of this paper is organized as follows. Section II elaborates on the CCOPF formulation for UEHN. The solution strategy to CCOPF is described in Section III. Section IV illustrates the performance of the proposed chance-constrained framework. The paper is concluded in Section V.

II. CCOPF FORMULATION FOR UEHN

In this section, an UEHN model with nonlinear power and heat flow equations is introduced. Then, the CCOPF formulation for UEHN is developed to address uncertain PV outputs and loads.

A. UEHN Model

UEHN consist of EPS and DHS via CHP units, which generate electricity and heat simultaneously. CHP units, acting as linkages between EPS and DHS, allow the energy flows between the two networks.

1) *Power Flow Formulation of EPS*: Alternating current (AC) nodal power flow balance models are as follows:

$$P_{G,i} - P_{L,i} = V_i \sum_{j \in i} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}), \quad \forall i, j \in \Omega_P \quad (1)$$

$$Q_{G,i} - Q_{L,i} = V_i \sum_{j \in i} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}), \quad \forall i, j \in \Omega_P \quad (2)$$

where $P_{G,i}$ and $Q_{G,i}$ refer to the active and reactive power generation of bus i , while $P_{L,i}$ and $Q_{L,i}$ indicate the active and reactive power loads at the bus respectively. V_i indicates the voltage magnitude of bus i and V_j indicates that of bus j . θ_{ij} is the voltage phase angle difference between bus i and bus j . G_{ij} and B_{ij} are the equivalent conductance and admittance of the branch respectively while Ω_P is the index set of buses in the EPS.

2) *Heat Flow Formulation of DHS*: DHS, which is primarily composed of heat sources, district heating networks and heat consumers, undertakes the role of heat production, transmission and exchange. District heating networks usually consist of supply pipelines and return pipelines. Heat is transferred from heat sources to terminal heat consumers by hot water or hot water vapor in pipelines (currently, hot water is most commonly used in China).

The pipe mass flow rate within each pipeline of district heating networks is determined by the hydraulic model. The hydraulic model can be expressed by the continuity of flow equation, loop pressure equation, and head loss equation, namely:

$$\mathbf{A}_H \mathbf{m} = \mathbf{m}_q \quad (3)$$

$$\mathbf{B}_H \mathbf{h}_f = 0 \quad (4)$$

$$\mathbf{h}_f = \mathbf{K} \mathbf{m} |\mathbf{m}| \quad (5)$$

where $\mathbf{A}_H$ represents the node-branch network incidence matrix; $\mathbf{m}$ indicates the pipe mass flow rate; $\mathbf{m}_q$ is the mass flow rate injected from a source or discharged to a load; $\mathbf{B}_H$ refers to the loop-branch network incidence matrix; $\mathbf{h}_f$ is the head loss caused by friction; $\mathbf{K}$ expresses the resistance coefficients of each pipe, which, to a large extent, depends on the diameter of the pipeline. The details are described in [6], [7].

The node temperature of district heating networks is obtained by the thermal model which primarily involves the following three types of temperatures. The supply temperature T_s indicates the temperature of the flow before being injected into the node. The outlet temperature T_0 is defined as the temperature of the flow at the outlet of each node before mixing in the return pipelines. While the return temperature T_r expresses the temperature of the flow after flowing out of the node and mixing with that of other pipelines into the return pipelines. The thermal model can be expressed by a nodal heat energy equation, pipeline temperature drop equation and nodal mixture temperature equation, namely:

$$H = C_p m_q (T_s - T_0) \quad (6)$$

$$T_{\text{end}} = (T_{\text{start}} - T_a) e^{-\frac{\lambda L}{C_p m}} + T_a \quad (7)$$

$$\left(\sum m_{\text{out}}\right) T_{\text{out}} = \sum (m_{\text{in}} T_{\text{in}}) \quad (8)$$

where, H indicates the heat energy consumed or supplied at each node; C_p is the specified heat capacity of water; T_{start} and T_{end} indicates the temperature at the start and the end of a pipeline respectively; T_a refers to the ambient temperature; λ is the overall heat transfer coefficient of each pipeline per unit length and L is the length of the pipeline. m_{out} and m_{in} indicate the mass flow rate within a pipeline leaving and coming into a node respectively. T_{in} represents the temperatures of flow at the end of an incoming pipeline and T_{out} is the mixture temperature of the node.

3) *CHP Units*: CHP units deliver electricity to the EPS, as well as heat to the DHS. Typically, in CHP units, heat that is normally wasted in conventional power generation is recovered as useful energy for heating, which avoids the losses that would otherwise be incurred from separate generation of heat and power. According to the different steam utilization methods, CHP units can be classified into back-pressure CHP units and extraction-condensing CHP units [11], [17]. There is a feasible region for electric power and heat outputs of a CHP unit, as shown in Fig. 1.

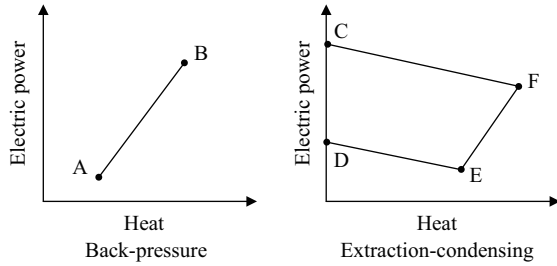


Fig. 1. Feasible region of combined heat and power (CHP) units.

The electric power and heat outputs of both types of CHP units can be formulated uniformly by the coordinates of the extreme points, namely:

$$P_{\text{CHP},i} = \sum_{k \in \Omega_{\text{CHP},i}} \eta_{i,k} P_{\text{CHP},i,k} \quad (9)$$

$$H_{\text{CHP},i} = \sum_{k \in \Omega_{\text{CHP},i}} \eta_{i,k} H_{\text{CHP},i,k} \quad (10)$$

$$0 \leq \eta_{i,k} \leq 1 \quad \sum_{k \in \Omega_{\text{CHP},i}} \eta_{i,k} = 1, \quad \forall i \in \Omega_{\text{CHP}} \quad (11)$$

where $P_{\text{CHP},i}$ and $H_{\text{CHP},i}$ are the electric power and heat outputs of the CHP unit; $P_{\text{CHP},i,k}$ and $H_{\text{CHP},i,k}$ are the corresponding electric power and heat outputs at the extreme point k of the CHP unit i ; $\eta_{i,k}$ is the coefficient of the CHP unit i at the extreme point k ; $\Omega_{\text{CHP},i}$ is the index set of extreme points of the CHP unit i and Ω_{CHP} is the index set of the CHP units.

B. Uncertainties of PV Outputs and Loads

This paper focuses on the uncertainties of sources (such as PV outputs, electricity loads and heat loads). The variations of loads are typically modeled as correlated normal distributions,

and the uncertainties of PV outputs are represented via Beta distributions with known correlation matrices.

1) *Probability Density Function (PDF) of PV Outputs*: PV outputs, characterized by their strong fluctuant and intermittent nature, satisfy Beta distributions [22]:

$$f(P_{\text{PV}}) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{P_{\text{PV}}}{P_{\text{PV,max}}}\right)^{\alpha-1} \cdot \left(1 - \frac{P_{\text{PV}}}{P_{\text{PV,max}}}\right)^{\beta-1} \quad (12)$$

where α and β are the shape parameters of the Beta distribution; P_{PV} and $P_{\text{PV,max}}$ are the actual power outputs and the installed capacities of the PV power plants.

2) *PDF of Electricity Loads and Heat Loads*:

$$f(P_{\text{load}}) = \frac{1}{\sqrt{2\pi}\sigma_{P_{\text{load}}}} \exp\left(-\frac{(P_{\text{load}} - \mu_{P_{\text{load}}})^2}{2\sigma_{P_{\text{load}}}^2}\right) \quad (13)$$

$$f(H_{\text{load}}) = \frac{1}{\sqrt{2\pi}\sigma_{H_{\text{load}}}} \exp\left(-\frac{(H_{\text{load}} - \mu_{H_{\text{load}}})^2}{2\sigma_{H_{\text{load}}}^2}\right) \quad (14)$$

where $\mu_{P_{\text{load}}}$ and $\sigma_{P_{\text{load}}}$ are the expected values (EVs) and standard deviations (SDs) of the electricity loads P_{load} ; $\mu_{H_{\text{load}}}$ and $\sigma_{H_{\text{load}}}$ are the EVs and SDs of the heat loads H_{load} .

A constant power factor control strategy is adopted in the PV outputs model and the electricity and heat loads model.

C. CCOPF Formulation for UEHN

The CCOPF formulation for UEHN minimizes the total operational costs for both EPS and DHS while satisfying steady-state operational constraints, including deterministic constraints and chance constraints.

1) *Objective Function*:

$$\min f \sum_{i \in \Omega_{\text{CHP}}} C_{\text{CHP},i}(P_{\text{CHP},i}, H_{\text{CHP},i}) \quad (15)$$

where f is the total operational costs; $C_{\text{CHP},i}$ is the quadric cost function of the CHP unit i [17], [23]:

$$C_{\text{CHP},i}(P_{\text{CHP},i}, H_{\text{CHP},i}) = \mu_1 + \mu_2 P_{\text{CHP},i} + \mu_3 H_{\text{CHP},i} + \mu_4 (P_{\text{CHP},i})^2 + \mu_5 (H_{\text{CHP},i})^2 + \mu_6 P_{\text{CHP},i} H_{\text{CHP},i} \quad (16)$$

where μ_1 – μ_6 are the cost coefficients of the CHP unit i .

2) *Deterministic Constraints*: As shown in (1)–(11), deterministic equality constraints are obtained. Equations (1) and (2) denote AC nodal power flow balance models. Nodal heat flow balance is expressed by (6). The hydraulic model is adopted in (3)–(5) and the thermal model in (7)–(8). CHP unit constraints are denoted by (9)–(11).

The deterministic inequality constraints can be expressed as follows:

$$P_{\text{CHP},i,\min} \leq P_{\text{CHP},i} \leq P_{\text{CHP},i,\max} \quad (17)$$

$$Q_{\text{CHP},i,\min} \leq Q_{\text{CHP},i} \leq Q_{\text{CHP},i,\max} \quad (18)$$

$$H_{\text{CHP},i,\min} \leq H_{\text{CHP},i} \leq H_{\text{CHP},i,\max} \quad (19)$$

where $P_{\text{CHP},i,\min}$ and $P_{\text{CHP},i,\max}$, $Q_{\text{CHP},i,\min}$ and $Q_{\text{CHP},i,\max}$, $H_{\text{CHP},i,\min}$ and $H_{\text{CHP},i,\max}$ are the upper and lower bounds of the active electricity outputs, reactive electricity outputs, and heat outputs of the CHP unit i respectively.

3) *Chance Constraints*: While considering uncertain sources, the state variables (usually consist of voltage magnitudes in EPS, pipe mass flow rates, supply temperatures and return temperatures in DHS) are all random variables, it is desired that the original constraints should be tenable at a certain confidence level, namely chance constraints:

$$\Pr\{V_i \leq V_{i,\max}\} \geq p_{V,\max}, \Pr\{V_i \geq V_{i,\min}\} \geq p_{V,\min} \quad (20)$$

$$\Pr\{m_i \leq m_{i,\max}\} \geq p_{m,\max}, \Pr\{m_i \geq m_{i,\min}\} \geq p_{m,\min} \quad (21)$$

$$\Pr\{T_{s,i} \leq T_{s,i,\max}\} \geq p_{T_s,\max}, \Pr\{T_{s,i} \geq T_{s,i,\min}\} \geq p_{T_s,\min} \quad (22)$$

$$\Pr\{T_{r,i} \leq T_{r,i,\max}\} \geq p_{T_r,\max}, \Pr\{T_{r,i} \geq T_{r,i,\min}\} \geq p_{T_r,\min} \quad (23)$$

where $\Pr\{\cdot\}$ is the probability measure of the chance constraint; the subscripts “max” and “min” represent the upper and lower bounds of the corresponding variables in the formulation; p_V , p_m , p_{T_s} , and p_{T_r} are the preset confidence levels of the corresponding variables.

III. SOLUTION STRATEGY

A. LHS MCS Approach

The LHS MCS approach, which is a combination of LHS MCS and Nataf transformation, is adopted to calculate the probabilistic power and heat flows of UEHN. LHS is a type of stratified method [24], [25], which can cover larger sampling space of input random variables with a smaller sample sizes compared with simple random sampling. These samplings are rearranged to keep the correlations between the input random variables based on the Nataf transformation. The LHS MCS approach has the following advantages: 1) LHS MCS approach can provide the overall information of the output random variables; 2) LHS MCS approach can address correlations of input random variables and is almost unconstrained by the types of the probability distributions of the input random variables; 3) LHS MCS approach is simple to implement with good robustness and high precision.

The basic principles of the Nataf transformation are introduced as follows:

Let C_X be the correlation matrix of the n dimension input random variable. The n dimension standard normal distribution random variable $Y = (y_1, y_2, \dots, y_n)^T$ with the correlation matrix C_Y are introduced. According to the equal probability principle:

$$y_k = \psi_k^{-1}(F_k(x_k)), \quad \forall k \in \Omega_k \quad (24)$$

where F_k is the cumulative distribution function (CDF) of x_k ; ψ_k is the CDF of y_k ; Ω_k is the index set of variables.

A lower triangular matrix D is obtained by the Cholesky decomposition of the positive definite matrix C_Y . The n dimension independent standard normal variable $Z = (z_1, z_2, \dots, z_n)^T$ is:

$$Z = D^{-1}Y. \quad (25)$$

The above-mentioned Nataf transformation realizes the conversion from the input variable space X , to the correlated standard normal space Y by (24), and then to the independent

standard normal space Z via (25). The key of the transformation is to obtain C_Y with given C_X . This paper combines the two-dimensional Nataf transformation with the Gauss-Hermite integration to obtain the correlation matrix C_Y , which has relatively high precision and avoids the calculation of infinite integration; detailed information can be found in [26].

B. Distributions of State Variables

In this section, to obtain the numerical expression of the state variable distributions, first, the various order cumulants are calculated based on the samplings of the state variable distributions acquired from the LHS MCS approach, then, the numerical expression of the state variable distributions are obtained through the Cornish-Fisher series approximation.

1) *Calculation of Cumulant*: Considering that the distribution types of the state variables are unknown, to the best of the authors' knowledge, it is hard to compute the cumulant with conventional numerical methods.

This paper calculates the v order origin moments χ_v according to the state variable distribution samplings $S_{m \times N} = (s_1, s_2, \dots, s_N)$. Each order cumulant of the state variables is obtained according to the relationship between the cumulant κ_v and the origin moment χ_v in [27].

$$\chi_v = \frac{1}{N} \sum_{i=1}^N s_i^v, \quad v = 1, 2, \dots \quad (26)$$

$$\begin{cases} \kappa_1 = \chi_1 \\ \kappa_{v+1} = \chi_{v+1} - \sum_{j=1}^v C_v^j \chi_j \kappa_{v-j+1} \end{cases} \quad (27)$$

where C_v^j represents the combination number.

2) *Cornish-Fisher Expansion*: Currently, the Gram-Charlier series and Cornish-Fisher series are commonly employed to fit the distributions based on cumulants, but it has been identified in [21], [28] that the Cornish-Fisher series performs better than the Gram-Charlier series when non-Gaussian distributions, such as Beta distributions, are considered. Therefore' in this paper, up to the 5th order Cornish-Fisher series are employed to approximate the quantile function R of the state variables:

$$\begin{aligned} R(\omega) = & \xi(\omega) + \frac{\xi^2(\omega) - 1}{6} g_3 + \frac{\xi^3(\omega) - 3\xi(\omega)}{24} g_4 - \\ & \frac{2\xi^3(\omega) - 5\xi(\omega)}{36} g_3^2 + \frac{\xi^4(\omega) - 6\xi^2(\omega) + 3}{120} g_5 - \\ & \frac{\xi^4(\omega) - 5\xi^2(\omega)}{24} g_2 g_3 + \frac{12\xi^4(\omega) - 53\xi^2(\omega)}{324} g_3^3 + \dots \end{aligned} \quad (28)$$

where $R(\omega)$ is the quantile of the state variable at ω ; $\xi(\omega)$ represents the inverse function of the standard normal distribution; g_v is the standardized v order cumulant, $g_v = \kappa_v / \sqrt{\kappa_2^v}$.

C. Heuristic Algorithm

To solve the CCOPE, this paper converts all the chance constraints into equivalent deterministic constraints and adopts a heuristic algorithm presented in [21], [29]. The chance constraints in (20)–(23) can be expressed in a unified form

as follows:

$$\begin{cases} \Pr \{x_i \leq x_{i,\max}\} \geq p_{x_i,\max} \\ \Pr \{x_i \geq x_{i,\min}\} \geq p_{x_i,\min} \end{cases} \quad (29)$$

Let F_{x_i} be the CDF of the state variable x_i , equation (29) can be transformed to be:

$$\begin{cases} F_{x_i}(x_{i,\max}) \geq p_{x_i,\max} \\ F_{x_i}(x_{i,\min}) \leq 1 - p_{x_i,\min} \end{cases} \quad (30)$$

As presented in Section III-B, the Cornish-Fisher series can provide an approximation of the quantile function of x_i based on the cumulant. Because the quantile function R_{x_i} is actually the inverse function of F_{x_i} which is monotone increasing, so the chance constraints in (32) can be equivalently converted to the following form:

$$\begin{cases} R_{x_i}(p_{x_i,\max}) \leq x_{i,\max} \\ R_{x_i}(1 - p_{x_i,\min}) \geq x_{i,\min} \end{cases} \quad (31)$$

The chance constraints are verified by comparing the quantile and constraint bounds of the state variables. When a violation occurs, we adjust the constraint bound by the following heuristic algorithm.

If $R_{x_i}(p_{x_i,\max}) > x_{i,\max}$, we adjust the upper bound of the state variable x_i :

$$\hat{x}_{i,\max} = \hat{x}_{i,\max} \max \left\{ 1 - \frac{R_{x_i}(p_{x_i,\max}) - \hat{x}_{i,\max}}{\hat{x}_{i,\max}}, 1 - \delta \right\}. \quad (32)$$

If $R_{x_i}(1 - p_{x_i,\min}) < x_{i,\min}$, we adjust the lower bound of the state variable x_i :

$$\hat{x}_{i,\min} = \hat{x}_{i,\min} \min \left\{ 1 + \frac{\hat{x}_{i,\min} - R_{x_i}(1 - p_{x_i,\min})}{\hat{x}_{i,\min}}, 1 + \delta \right\} \quad (33)$$

where $\hat{x}_{i,\max}$ and $\hat{x}_{i,\min}$ represent the upper and lower bounds of the state variables during the calculation; the initial values are $x_{i,\max}$ and $x_{i,\min}$; the adjustment coefficient δ is applied to prevent an unreasonable adjustment resulting from a severe constraint violation, δ can be set to be 5% to 20% in [21].

D. Flowchart of Solution Strategy

The computational procedure of the solution strategy can be concluded in Fig. 2 and the main steps are as follows:

Step 1: Input data. Namely, network topology information, the CDF and correlation matrix of the random variables, adjustment coefficient and confidence level, etc.

Step 2: Deterministic optimization. Based on the primal-dual decomposed interior point method, an optimal scheduling strategy that corresponds to the deterministic case is obtained with the EVs of the input random variables.

Step 3: Probabilistic energy flow analysis considering correlations. Under the optimal scheduling strategy in Step 2, the LHS MCS approach is employed to perform a probabilistic power and heat flow analysis.

Step 4: Distribution calculation. The Cornish-Fisher series is employed to approximate the quantile of the state variables based on the cumulant, accordingly, the constraint violation probability can be obtained.

Step 5: Chance constraint verification. Compare the constraint violation probabilities with the allowed confidence level. If all the state variables satisfy the chance constraints, stop the iteration and the final optimal scheduling strategy is obtained, which includes the total operational cost, the outputs of the CHP units and the distributions of the state variables etc.

If a violation exists, adjust the corresponding upper and lower bounds of the chance constraint according to a heuristic algorithm and return to Step 2. This process repetitively runs until no violation of the constraints occur.

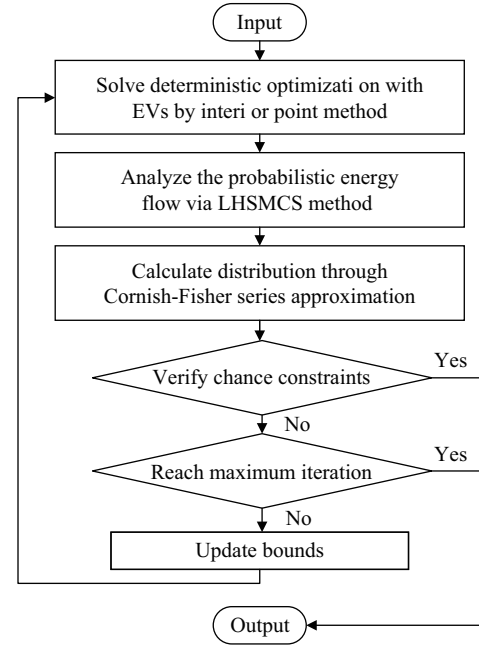


Fig. 2. Flow chart of CCOPF.

IV. CASE STUDIES

A. Test System Description

An UEHN was analyzed to illustrate the performance of the proposed chance-constrained framework. The topology of the UEHN is shown in Fig. 3. The UEHN consists of the following three parts: 1) A modified IEEE 33-bus system with PV power [30], [31]; 2) A 32-node DHS at Barry Island [6], [7]; 3) CHP units. The interdependence between EPS and DHS is presented in Table I.

TABLE I
INTERDEPENDENCE BETWEEN EPS AND DHS

CHP Unit Number	Type	EPS Bus	DHS Node
CHP1	Back-pressure	1	31
CHP2	Extraction-condensing	18	1
CHP3	Back-pressure	33	32

The UEHN has a total active electricity load of 3.7150 MW, a total reactive electricity load of 2.30 MV and a total heat load of 2.164 MW. The SDs of the electricity and heat loads are equal to 10% of their EVs. Four PV power plants with the same installed capacity of 0.5 MW are assumed to be

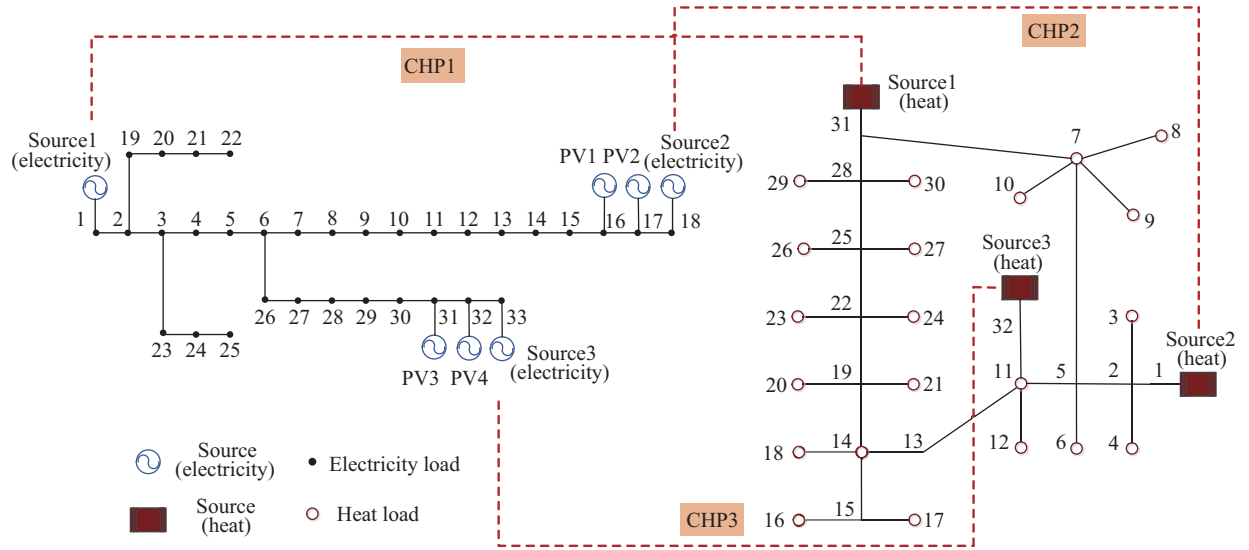


Fig. 3. An UEHN comprised of a modified IEEE 33-bus system and a 32-node DHS at Barry Island.

connected to bus 16, 17, 31, and 32 of the IEEE 33-bus system. The PV outputs satisfy the Beta distribution with shape parameters α and β both set to 12 in this paper.

Three types of correlations are considered, i.e. the correlations of the electricity-electricity loads and heat-heat loads are both set to 0.7; the correlations of the electricity-heat is set to 0.5; the correlations matrix of the PV outputs of the four PV power plants is given in the following [30], [32]:

$$C_{PV} = \begin{bmatrix} 1 & 0.5 & 0.4 & 0.4 \\ 0.5 & 1 & 0.4 & 0.4 \\ 0.4 & 0.4 & 1 & 0.5 \\ 0.4 & 0.4 & 0.5 & 1 \end{bmatrix}. \quad (34)$$

The operational cost coefficients μ_1 – μ_6 are presented in Table II. Moreover, the upper and lower bounds of the voltage magnitudes are set to 1.05 and 0.95. The maximum mass flow rate in DHS is 5 kg/s. The upper and lower bounds of the supply temperature are 60°C and 56°C while those of the return temperature are 20°C and 16°C. The confidence level of the chance constraints are all set to 95%.

TABLE II
COST COEFFICIENT

Cost Coefficient	Value	Cost Coefficient	Value
μ_1	2650	μ_4	0.0345
μ_2	14.5	μ_5	0.03
μ_3	4.2	μ_6	0.031

B. The Performance of the Solution Strategy

The simulation is performed on a 64-bit PC with 3.2 GHz CPU and 4.0 GB RAM in a MATLAB environment. The required computation time is 56 s. The CCOPF solution strategy converges in 6 iterations, and the objective is \$8,000.8.

Table III demonstrates the total operational cost of the UEHN and the number of variables that violate the chance constraints during each of the iterations. As shown in Table III, due to the influence of uncertain sources, the total

operational cost is slightly larger than that of the deterministic optimization with all random variables set at their EVs (i.e. the operational cost after 1st iteration).

TABLE III
ITERATION INFORMATION DURING CCOPF SOLUTION

Iteration	Total Cost (\$)	Number of Constraint Violation	
		EPS	DHS
1	7999.9	7	3
2	8000.2	5	2
3	8000.5	2	1
4	8000.7	1	1
5	8000.8	0	1
6	8000.8	0	0

C. Correlation

Significant correlations exist between the adjacent PV power plants. There are correlations between the electricity and heat loads in the same region. To study the impact of these correlations on system operations, the following three typical scenarios are implemented:

1) Scenario 1: Take into account all three types of correlations, as illustrated in Section IV-A.

2) Scenario 2: Consider the correlations between the loads, that is, the correlations of the electricity-electricity loads, the heat-heat loads and the electricity-heat loads.

3) Scenario 3: Irrespective of the correlation.

In Figs. 4(a)–(c), the probability distributions of the voltage magnitude at bus 31 in the EPS, the pipe mass flow rate within pipeline 12 and the supply temperature at node 6 in the DHS under different scenarios are compared.

As shown in Fig. 4(a), considering correlations of uncertain sources intensifies the volatility of the state variables in the EPS. With more variables considered correlated, the state variables in the EPS possess stronger volatility. This is attributed to the fact that the correlations of uncertain sources, especially the PV outputs, have made it much more likely to suffer peaks

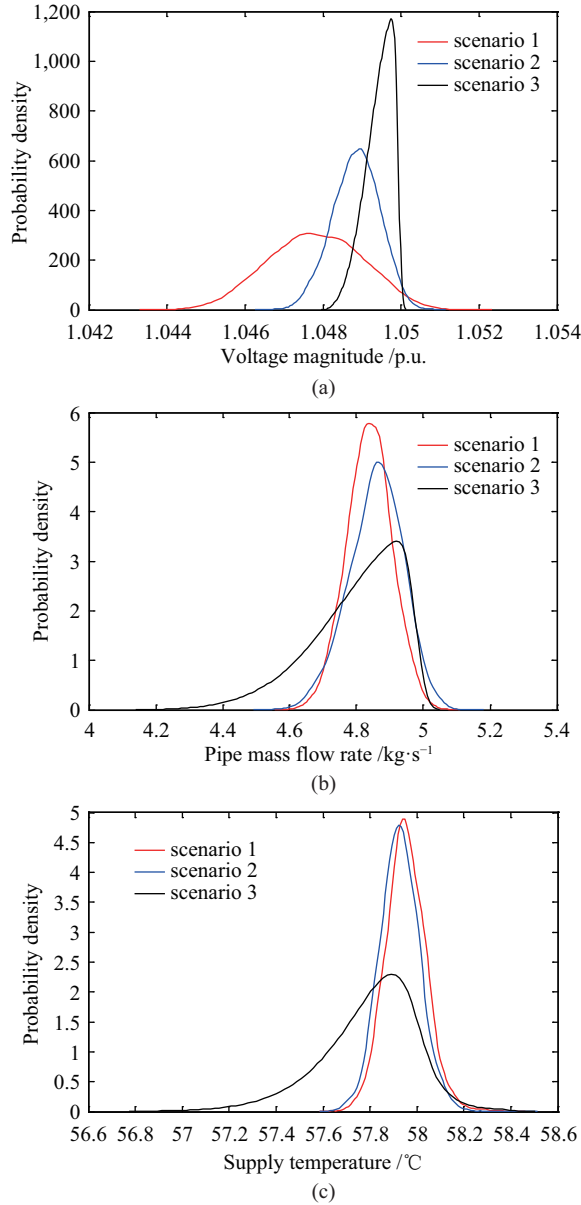


Fig. 4. Probability density curves under 3 scenarios. (a) Voltage magnitude at bus 31. (b) Pipe mass flow rate within pipeline 12. (c) Supply temperature at node 6.

and valleys at the same time, leading to a larger volatility. In comparison, a decreased volatility of the state variables in the DHS can be observed from Figs. 4(b) and 4(c) when correlated uncertain sources are considered (scenarios 1 and 2).

Figs. 4(a)–4(c) indicate that consideration of the correlations between uncertainty sources greatly impacts the constraint violation probabilities of state variables in both the EPS and DHS. In this regard, the necessity of inclusion of correlated uncertain sources in the proposed CCOPF is demonstrated.

D. Confidence Level

CCOPF requires that the chance constraints of all state variables both in EPS and DHS are satisfied at a certain confidence level, and operational performance varies in different confidence levels. We discuss the impact of confidence levels

under the following three scenarios:

- 1) Scenario 1: Confidence level is set to 95%.
- 2) Scenario 2: Confidence level is set to 85%.
- 3) Scenario 3: Deterministic optimization irrespective of chance constraints.

In Figs. 5(a)–(b), the probability distributions of the voltage magnitude at bus 31 in EPS and the pipe mass flow rate within pipeline 13 in the DHS under different scenarios are compared.

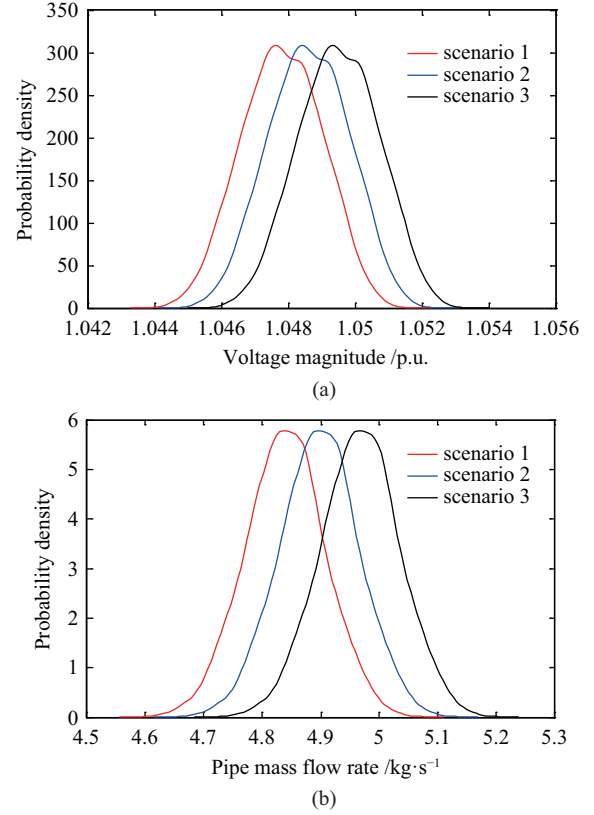


Fig. 5. Probability density curves under 3 scenarios. (a) Voltage magnitude at bus 31. (b) Pipe mass flow rate within pipeline 13.

As seen from the results, in the case of the deterministic optimization (Scenario 3), the EVs of the state variables are very close to upper and lower bounds. For example, in Fig. 5(a), the likelihood that the voltage magnitude at bus 31 exceeds its upper limit is up to 40% under scenario 3. Under these circumstances, a sudden load or PV output deviation will lead part or all of the variables to break their bounds. When the confidence level is set to 85% (Scenario 2), the insecurity risk is decreased. When you increase the confidence level to 95% (Scenario 1), the insecurity risk is further reduced. This is because the higher the confidence level, the narrower the fluctuation ranges of the state variables in the optimization. Meanwhile, the total operational costs increase with the improvement in the confidence level. The overall operational costs from Scenario 1 to Scenario 3 are ranging from \$8,000.8, \$8,000.3 to \$7,999.9. When the confidence level increases, larger amounts of power should be generated in reaction to a sudden load or PV power deviation. This results in a higher operational cost.

As verified in Figs. 5(a)–(b), the CCOPF could significantly

reduce insecurity risk, and the higher the confident level is, the lower the risk of insecurity and the higher the operational costs of the UEHN will be.

V. CONCLUSION

We propose a CCOPF formulation for UEHN in this paper. A solution strategy that combines the LHSMCS approach and a heuristic algorithm is specially designed. The proposed framework can deal with not only the uncertainties with non-Gaussian distributions (i.e., PV outputs), but also the correlations between them. Case studies verify the satisfying convergence of the framework and demonstrate the necessity of inclusion of correlated uncertain sources in the proposed CCOPF. Meanwhile, the impacts of different confidence levels of CCOPF on the system operations are also investigated. Finally, we need to clarify that the obtained CCOPF solution may not be globally optimal, which requires further investigation in the future.

REFERENCES

- [1] M. O'Malley and B. Kroposki, "Energy comes together: the integration of all systems [Guest Editorial]," *IEEE Power and Energy Magazine*, vol. 11, no. 5, pp. 18–23, Sep./Oct. 2013.
- [2] J. Z. Wu, J. Y. Yan, H. J. Jia, N. Hatziaargyriou, N. Djilali, and H. B. Sun, "Integrated energy systems," *Applied Energy*, vol. 167, pp. 155–157, Apr. 2016.
- [3] S. Y. Wu, P. Wang, J. Yang, Z. N. Li, and M. Ouyang, "Review on interdependency modeling of integrated energy system," in *2017 IEEE Conference on Energy Internet and Energy System Integration (EI2)*, Beijing, China, 2017, pp. 1–6.
- [4] J. Z. Wu, "Drivers and state-of-the-art of integrated energy systems in Europe," *Automation of Electric Power Systems*, vol. 40, no. 5, pp. 1–7, Mar. 2016.
- [5] W. L. Wang, D. Wang, H. J. Jia, Z. Y. Chen, B. Q. Guo, H. M. Zhou, and M. H. Fan, "Review of steady-state analysis of typical regional integrated energy system under the background of energy internet," *Proceedings of the CSEE*, vol. 36, no. 12, pp. 3292–3306, Jun. 2016.
- [6] X. Z. Liu, "Combined analysis of electricity and heat networks," Ph.D. dissertation, Institute of Energy, Cardiff University, Wales, UK, 2013.
- [7] X. Z. Liu, J. Z. Wu, N. Jenkins, and A. Bagdanavicius, "Combined analysis of electricity and heat networks," *Applied Energy*, vol. 162, pp. 1238–1250, Jan. 2016.
- [8] A. Shabanpour-Haghighi and A. R. Seifi, "An integrated steady-state operation assessment of electrical, natural gas, and district heating networks," *IEEE Transactions on Power Systems*, vol. 31, no. 5, pp. 3636–3647, Sep. 2016.
- [9] Z. G. Pan, Q. L. Guo, and H. B. Sun, "Interactions of district electricity and heating systems considering time-scale characteristics based on quasi-steady multi-energy flow," *Applied Energy*, vol. 167, pp. 230–243, Apr. 2016.
- [10] J. N. Dong, H. B. Sun, Q. L. Guo, and Z. G. Pan, "State estimation for combined electricity and heat networks," *Power System Technology*, vol. 40, no. 6, pp. 1635–1641, Jun. 2016.
- [11] X. Y. Chen, C. Q. Kang, M. O'Malley, Q. Xia, J. H. Bai, C. Liu, R. F. Sun, W. Z. Wang, and H. Li, "Increasing the flexibility of combined heat and power for wind power integration in China: modeling and implications," *IEEE Transactions on Power Systems*, vol. 30, no. 4, pp. 1848–1857, Jul. 2015.
- [12] Z. G. Li, W. C. Wu, M. Shahidehpour, J. H. Wang, and B. M. Zhang, "Combined heat and power dispatch considering pipeline energy storage of district heating network," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 1, pp. 12–22, Jan. 2016.
- [13] J. H. Li, J. K. Fang, Q. Zeng, and Z. Chen, "Optimal operation of the integrated electrical and heating systems to accommodate the intermittent renewable sources," *Applied Energy*, vol. 167, pp. 244–254, Jan. 2016.
- [14] Y. H. Dai, L. Chen, Y. Min, F. Xu, K. Y. Hou, and Y. Zhou, "Optimal dispatch for joint operation of wind farm and combined heat and power plant with thermal energy storage," *Proceedings of the CSEE*, vol. 37, no. 12, pp. 3470–3479, Jun. 2017.
- [15] C. H. Lin, W. C. Wu, B. M. Zhang, and Y. Sun, "Decentralized solution for combined heat and power dispatch through benders decomposition," *IEEE Transactions on Sustainable Energy*, vol. 8, no. 4, pp. 1361–1372, Oct. 2017.
- [16] M. K. Bai, Y. Wang, W. Tang, C. Wu, and B. Zhang, "Day-ahead optimal dispatching of regional integrated energy system based on interval linear programming," *Power System Technology*, vol. 41, no. 12, pp. 3963–3970, Dec. 2017.
- [17] A. Charnes and W. W. Cooper, "Chance-constrained programming," *Management Science*, vol. 6, no. 1, pp. 73–79, Oct. 1959.
- [18] H. Zhang and P. Li, "Chance constrained programming for optimal power flow under uncertainty," *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 2417–2424, Nov. 2011.
- [19] Z. Wu, P. L. Zeng, X. P. Zhang, and Q. Y. Zhou, "A solution to the chance-constrained two-stage stochastic program for unit commitment with wind energy integration," *IEEE Transactions on Power Systems*, vol. 31, no. 6, pp. 4185–4196, Nov. 2016.
- [20] H. Yu, C. Y. Chung, K. P. Wong, and J. H. Zhang, "A chance constrained transmission network expansion planning method with consideration of load and wind farm uncertainties," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1568–1576, Nov. 2009.
- [21] G. Li and X. P. Zhang, "Stochastic optimal power flow approach considering correlated probabilistic load and wind farm generation," in *IET Conference on Reliability of Transmission and Distribution Networks*, London, UK, 2011, pp. 1–7.
- [22] C. Wan, J. Zhao, Y. H. Song, Z. Xu, J. Lin, and Z. C. Hu, "Photovoltaic and solar power forecasting for smart grid energy management," *CSEE Journal of Power and Energy Systems*, vol. 1, no. 4, pp. 38–46, Dec. 2015.
- [23] T. Guo, M. I. Henwood, and M. van Ooijen, "An algorithm for combined heat and power economic dispatch," *IEEE Transactions on Power Systems*, vol. 11, no. 4, pp. 1778–1784, Nov. 1996.
- [24] Y. Chen, J. Y. Wen, and S. J. Cheng, "Probabilistic load flow method based on Nataf transformation and Latin Hypercube Sampling," *IEEE Transactions on Sustainable Energy*, vol. 4, no. 2, pp. 294–301, Apr. 2013.
- [25] Z. N. Wei, S. Chen, G. Q. Sun, D. Wang, Y. H. Sun, and H. X. Zang, "Probabilistic available transfer capability calculation considering static security constraints and uncertainties of electricity-gas integrated energy systems," *Applied Energy*, vol. 167, pp. 305–316, Apr. 2016.
- [26] P. L. Liu and A. Der Kiureghian, "Multivariate distribution models with prescribed marginals and covariances," *Probabilistic Engineering Mechanics*, vol. 1, no. 2, pp. 105–112, 1986.
- [27] J. Usaola, "Probabilistic load flow with correlated wind power injections," *Electric Power Systems Research*, vol. 80, no. 5, pp. 528–536, May 2010.
- [28] J. Usaola, "Probabilistic load flow with wind production uncertainty using cumulants and Cornish-Fisher expansion," *International Journal of Electrical Power & Energy Systems*, vol. 31, no. 9, pp. 474–481, Oct. 2009.
- [29] G. Q. Sun, Y. C. Li, S. Chen, Z. N. Wei, S. Chen, and H. X. Zang, "Dynamic stochastic optimal power flow of wind power and the electric vehicle integrated power system considering temporal-spatial characteristics," *Journal of Renewable & Sustainable Energy*, vol. 8, no. 5, pp. 053309, Sep. 2016.
- [30] C. Chen, W. C. Wu, B. M. Zhang, and H. B. Sun, "Probabilistic load flow of distribution network considering correlated photovoltaic power output," *Automation of Electric Power Systems*, vol. 39, no. 9, pp. 41–47, May 2015.
- [31] F. J. Ruiz-Rodriguez, J. C. Hernandez, and F. Jurado, "Probabilistic load flow for radial distribution networks with photovoltaic generators," *IET Renewable Power Generation*, vol. 6, no. 2, pp. 110–121, Mar. 2012.
- [32] S. Chen, Z. N. Wei, G. Q. Sun, K. W. Cheung, and Y. H. Sun, "Multi-linear probabilistic energy flow analysis of integrated electrical and natural-gas systems," *IEEE Transactions on Power Systems*, vol. 32, no. 3, pp. 1970–1979, May 2017.



Zhinong Wei received the B.S. degree from Hefei University of Technology, Hefei, China, in 1984, the M.S. degree from Southeast University, Nanjing, China, in 1987, and the Ph.D. degree from Hohai University, Nanjing, China, in 2004. He is now a Professor of electrical engineering with the College of Energy and Electrical Engineering in Hohai University, Nanjing, China. His research interests include power system state estimation, integrated energy systems, smart distribution systems, optimization and planning, load forecasting, and integration

of distributed generation into electric power systems.



Sheng Chen (S'15) received the B.S. degree from the College of Energy and Electrical Engineering, Hohai University, Nanjing, China, in 2014, where he is currently pursuing his Ph.D. degree. His research interests include economic dispatch and security analysis of integrated energy systems.



Juan Sun received the B.S. degree from the College of Energy and Electrical Engineering, Hohai University, Nanjing, China, in 2016, where she is currently pursuing her M.S. degree. Her research interests include economic dispatch and security analysis of integrated electricity-heat energy systems.



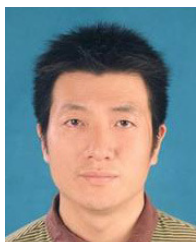
Side Zhang received his B.S. and M.S. degrees from the College of Energy and Electrical Engineering, Hohai University, Nanjing, China, in 2015 and 2018, respectively. His research interests include economic dispatch and security analysis of integrated electricity-gas energy systems.



Zhoujun Ma received the B.S. and M.S. degrees in Electrical Engineering from Southeast University and Shanghai Jiao Tong University, in 2009 and 2013, respectively. He is currently pursuing his Ph.D. degree at Hohai University. His research interests include power distribution dispatching, operation and control of power systems, etc.



Kwok W. Cheung (M'91–SM'98–F'14) received his Ph.D. in Electrical Engineering from Rensselaer Polytechnic Institute, Troy, NY, USA in 1991. He joined GE Grid Solutions (formerly ALSTOM Grid Inc.) in 1991 and is currently Director, R&D of Network Management Solutions focusing on innovation and technology. His current interests include electricity market design and implementation, smart grid, renewable energy integration, energy forecasting, power system stability and microgrid. Dr. Cheung has been a registered Professional Engineer of the State of Washington since 1994.



Guoqiang Sun (M'14) received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Hohai University, Nanjing, China, in 2001, 2005, and 2010, respectively. He was a Visiting Scholar at North Carolina State University, Raleigh, NC from 2015 to 2016. He is now an Associate Professor with the College of Energy and Electrical Engineering, Hohai University, Nanjing, China. His research interests include power system analysis and economic dispatch and optimal control of integrated energy systems.



Haixiang Zang received his B.S. and Ph.D. degrees in Electrical Engineering from Nanjing Normal University and Southeast University, in 2009 and 2014, respectively. He is now an Associate Professor with the College of Energy and Electrical Engineering, Hohai University, Nanjing, China. His research interests include generation of renewable energy, operation and control of power systems, etc.